2014. EVALUACIÓN #5

1) APLICANDO LA DEFINICIÓN DE TRANSFORMADA DE FOURIER, CALCULAR:

$$\mathcal{F}\{\cos(\omega_0 \tau) \cdot \mu(t)\} = F(j\omega)$$

Solución:

$$F(j\omega) = \int_{-\infty}^{+\infty} \cos(\omega_0 \tau)\, \mu(t) \cdot e^{-j\omega\tau} \cdot d\tau$$

$$F(j\omega) = \int_{-\infty}^{+\infty} \frac{e^{j\omega_0\tau} + e^{-j\omega_0\tau}}{2} \cdot \underbrace{\mu(t)}_{=0 \ \forall\ \tau<0} \cdot e^{-j\omega\tau} \cdot d\tau$$

$$F(j\omega) = \frac{1}{2}\int_{0}^{+\infty} \left(e^{j\omega_0 t} + e^{-j\omega_0\tau}\right) \cdot e^{-j\omega\tau} \cdot d\tau =$$

$$= \frac{1}{2}\int_0^{\infty} e^{-j\tau(\omega-\omega_0)}\, d\tau + \frac{1}{2}\int_0^{\infty} e^{-j\tau(\omega_0+\omega)}\, d\tau$$

$$= \frac{1}{2}\left\{ \frac{1}{j(\omega_0-\omega)} e^{-j\tau(\omega-\omega_0)} - \frac{1}{j(\omega_0+\omega)} e^{-j\tau(\omega_0+\omega)} \right\}_0^{\infty}$$

$$= \frac{1}{2} \frac{1}{j(\omega_0-\omega)} \left\{ e^{-\infty} - e^{0} \right\} - \frac{1}{2} \cdot \frac{1}{j(\omega_0+\omega)} \left\{ e^{-\infty} - 1 \right\}$$

$$\frac{1}{2}\left\{ \frac{-1}{j(\omega_0-\omega)} + \frac{1}{j(\omega_0+\omega)} \right\} = \frac{j\omega}{\omega_0^2 - \omega^2}$$

2) APLICANDO PROPIEDADES DE LA TRANSFORMADA DE FOURIER, CALCULAR LA TRANSFORMADA DE FOURIER

a) $x(t) = 5\,\mathrm{rect}\left(\frac{t+2}{4}\right) = 5\,\mathrm{rect}\left(\frac{1}{4}(t+2)\right)$

SEA:

$$f(t) = \mathrm{rect}(t) \Rightarrow F(j\omega) = \frac{2\,\mathrm{sen}(\omega T)}{\omega}$$

ENTONCES:

$$x(t) = 5\, f\left(\tfrac{1}{4}(t+2)\right) = 5\, e^{-j2\omega} \cdot 4 \cdot F(j4\omega)$$

$$X(j\omega) = 5 \cdot e^{-j2\omega} \cdot \cancel{4} \cdot \frac{2\,\mathrm{sen}(4\omega T)}{\cancel{4}\omega}$$

$$X(j\omega) = 10\, e^{-j2\omega} \cdot \frac{\mathrm{sen}(4\omega T)}{\omega}$$

b) $y(t) = \frac{d}{d\tau} e^{-j\omega_0(t-1)} \cdot \cos(\pi(\tau-1)) \cdot \mu(t-1)$

SEA:

$$f(t) = \cos(\pi\tau)\, \mu(t) \Rightarrow F(j\omega) = \frac{j\omega}{\pi^2 - \omega^2}$$

ENTONCES:

$$y(t) = \frac{d}{d\tau}\, e^{-j\omega_0(\tau-1)} \cdot f(\tau-1) \Rightarrow$$

$$Y(j\omega) = (j\omega) \cdot e^{-j\omega} \cdot \mathcal{F}\{ e^{-j\omega_0\tau} \cdot f(t) \}$$

$$Y(j\omega) = j\omega \cdot e^{-j\omega} \frac{j(\omega-\omega_0)}{\pi^2 - (\omega-\omega_0)^2}$$

$$Y(j\omega) = \frac{\omega \cdot e^{-j\omega}(\omega_0 - \omega)}{\pi^2 - (\omega-\omega_0)^2}$$

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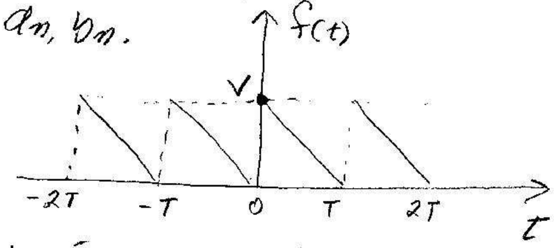
3) CALCULAR LOS COEFICIENTES DE LA SERIE DE LA SERIE DE FOURIER TRIGONOMETRICA-

a_0, a_n, b_n.



Solución:

CONVIENE ENFOCARSE EN EL RANGO $[-T, 0]$, PORQUE LA ECUACIÓN DE LA RECTA ES MÁS CORTA.

$$a_0 = \frac{1}{T}\int_{-T}^{0} f(t)\cdot d\tau = \frac{1}{T}\left(\frac{1}{2}\cdot V \cdot T\right) = \frac{V}{2}$$

$$a_n = \frac{2}{T}\int_{-T}^{0}\left(-\frac{V}{T}\tau\right)\cdot \cos(n\omega_0\tau)\cdot d\tau$$

NOTA:

$$\int x\cdot\cos(ax)\cdot dx = \frac{1}{a}x\,\mathrm{sen}\,ax + \frac{1}{a^2}\cos ax$$

ENTONCES:

$$a_n = \frac{-2V}{T^2}\left\{\frac{\tau}{n\omega_0}\cdot \mathrm{sen}(n\omega_0\tau) + \frac{1}{(n\omega_0)^2}\cos(n\omega_0\tau)\right\}_{-T}^{0}$$

$$a_n = \frac{-2V}{n\frac{2\pi}{T}T^2}\left[\tau\cdot \mathrm{Sen}\left(n\frac{2\pi}{T}\cdot\tau\right) + \frac{1}{n\frac{2\pi}{T}}\cdot\cos\left(n\frac{2\pi}{T}t\right)\right]_{-T}^{0}$$

$$a_n = \frac{-2V}{2\pi n T}\left[\left(0+\frac{T}{n2\pi}\right) - \left(-T\,\cancel{\mathrm{Sen}(-2\pi n)}^{\,0} + \frac{T}{n2\pi}\cos(2\pi n)\right)\right\}$$

$$a_n = \frac{-V}{\pi n T}\left\{\frac{T}{2\pi n} - \frac{T}{2\pi n}\cancel{\cos(2\pi n)}^{\,1}\right\} = 0$$

$$b_n = \frac{-2V}{T^2}\int_{-T}^{0}\tau\cdot \mathrm{Sen}(n\omega_0\tau)\cdot d\tau$$

$$b_n = \frac{-2V}{T^2}\left\{-\frac{\tau}{n\omega_0}\cdot\cos(n\omega_0\tau) + \frac{1}{n\omega_0}\int\cos(n\omega_0\tau)\cdot d\tau\right.$$

$$b_n = \frac{2V}{T^2}\left\{\frac{\tau}{n\omega_0}\cos(n\omega_0 t) - \frac{1}{(n\omega_0)^2}\,\mathrm{Sen}(n\omega_0\tau)\right\}_{-T}^{0}$$

$$b_n = \frac{2V}{T^2\cdot\frac{2\pi n}{T}}\cdot\left\{\tau\cdot\cos\left(n\frac{2\pi}{T}\cdot t\right) - \frac{1}{n\frac{2\pi}{T}}\cdot \mathrm{Sen}\left(n\frac{2\pi}{T}\cdot t\right)\right\}_{-T}^{0}$$

$$b_n = \frac{V}{T\pi n}\left\{[0-0] - \left[-T\cdot\cos(-2\pi n) - \frac{T}{2\pi n}\mathrm{Sen}(-2\pi n)\right]\right\}$$

$$b_n = \frac{TV}{T\pi n}\left\{\cancel{\cos(2\pi n)}^{\,1} + \cancel{\mathrm{Sen}(2\pi n)}^{\,0}\right\} = \frac{V}{n\pi}$$

c) CALCULAR $F(j\omega)\big|_{\omega=1+j}$ DONDE

$$f(t) = e^{-4\tau}\mu(t) + \cos(2\tau)\mu(t)$$

Solución:

$$F(j\omega) = \frac{1}{4+j\omega} + \frac{j\omega}{4-\omega^2}$$

$$F(j\omega_0) = \frac{1}{4+j(1+j)} + \frac{j(1+j)}{4-(1+j)^2} =$$

$$= \frac{1}{3+j} + \frac{-1+j}{4-1-2j+1} = \frac{1}{3+j} + \frac{-1+j}{4-2j}$$

$$= 0{,}3 - 0{,}1j + \frac{1{,}41\angle 135^\circ}{4{,}47\angle -26{,}5}$$

$$= 0{,}3 - 0{,}1j - 0{,}3 + 0{,}1j = 0\angle 0^\circ$$

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