EVALUACIÓN No 4

1) APLICANDO LA DEFINICIÓN DE TRANSFORMADA DE LAPLACE, CALCULAR $F(s)$.

$$f(t) = \begin{cases} 0 & \forall\ -\infty < t < 2 \\ e^{-at} & \forall\ 2 \leq t < \infty \end{cases}$$

Solución:

$$F(s) = \int_{-\infty}^{+\infty} f(t) \cdot e^{-st} \cdot dt = \int_{2}^{+\infty} e^{-at} \cdot e^{-st} \cdot dt$$

$$F(s) = \int_{2}^{\infty} e^{-t(s+a)} \cdot dt = \frac{-1}{s+a} e^{-t(s+a)} \Big|_{2}^{\infty}$$

$$F(s) = \frac{-1}{s+a} \left\{ e^{-\infty} - e^{-2(s+a)} \right\}$$

$$F(s) = \frac{e^{-2(s+a)}}{s+a}$$

2) RESOLVER LA SIG. ECUACIÓN DIFERENCIAL.

$$Y''(t) + 5Y'(t) - 6Y(t) = \frac{d}{dt} e^{-t} \cdot \mu(t)$$

CONDICIONES INICIALES = 0

$$s^2 Y(s) + 5sY(s) - 6Y(s) = s \cdot \frac{1}{s+1}$$

$$Y(s) = \frac{s}{(s+1)(s^2+5s-6)} =$$

$$= \frac{s}{(s+1)(s-1)(s+6)} = \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s+6}$$

$$A = \left[\frac{s}{(s-1)(s+6)} \right]_{s=-1} = 1/10$$

$$B = \frac{s}{(s+1)(s+6)} \Bigg|_{s=1} = 1/14 \ ; \ C = \frac{s}{(s+1)(s-1)} \Bigg|_{s=-6} = -6/35$$

ENTONCES:

$$Y(s) = \frac{1/10}{s+1} + \frac{1/14}{s-1} - \frac{6/35}{s+6}$$

$$y(t) = \left\{ \frac{1}{10} e^{-t} + \frac{1}{14} e^{t} - \frac{6}{35} e^{-6t} \right\} \mu(t)$$

3) CALCULAR $\mathcal{L}^{-1}\{X(s)\}$ DONDE

$$X(s) = \frac{s^2+s-0{,}5}{(s^2+s+0{,}5)(s+1)}$$

Solución:

$$X(s) = \frac{s^2+s-0{,}5}{(s+0{,}5+j0{,}5)(s+0{,}5-j0{,}5)(s+1)}$$

$$X(s) = \frac{N_1}{s+0{,}5+j0{,}5} + \frac{N_2}{s+0{,}5-j0{,}5} + \frac{N_3}{s+1}$$

$$N_1 = \frac{s^2+s-0{,}5}{(s+0{,}5-j0{,}5)(s+1)} \Bigg|_{s=-0{,}5-j0{,}5} = 1-j$$

$$N_2 = \frac{s^2+s-0{,}5}{(s+0{,}5+j0{,}5)(s+1)} \Bigg|_{s=-0{,}5+j0{,}5} = 1+j$$

$$N_3 = \frac{s^2+s-0{,}5}{(s^2+s+0{,}5)} \Bigg|_{s=-1} = -1$$

$$X(s) = \frac{1-j}{s+0{,}5+j0{,}5} + \frac{1-j}{s^2+0{,}5-j0{,}5} - \frac{1}{s+1}$$

$$X(t) = \left\{ e^{-t}(2\cos t - 2\operatorname{sen} t) - e^{-t} \right\} \mu(t)$$

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4) RESOLVER LA SIG. ECUACIÓN DIFERENCIAL, TODAS LAS CONDICIONES INICIALES SON IGUAL A CERO

$$y''_{(t)} + 5y'_{(t)} - 6y_{(t)} = \frac{d}{dt} e^{-t} \mu(t)$$

Solución:

APLICANDO TRANSF. DE LAPLACE

$$S^2 Y_{(s)} + 5S Y_{(s)} - 6 Y_{(s)} = S \cdot \frac{1}{S+6}$$

$$Y_{(s)} = \frac{S}{(S+6)(S^2+5S-6)} =$$

$$= \frac{S}{(S+6)^2(S-1)} = \frac{A}{S-1} + \frac{B}{(S+6)^2} + \frac{C}{S+6}$$

$$A = \left.\frac{S}{(S+6)^2}\right|_{S=1} = \frac{1}{(1+6)^2} = \frac{1}{14}$$

$$B = \left.\frac{S}{S-1}\right|_{S=-6} = \frac{-6}{-6-1} = 6/7$$

PARA $S=0$

$$\frac{S}{(S+6)^2(S-1)} = \frac{1/4}{S-1} + \frac{6/7}{(S+6)^2} + \frac{C}{S+6}$$

$$0 = -\frac{1}{4} + \frac{1}{42} + \frac{C}{6} \Rightarrow C = \frac{19}{14}$$

$$Y_{(s)} = \frac{1/14}{S-1} + \underbrace{\frac{6/7}{(S+6)^2}}_{Y_{1(S)}} + \frac{19/14}{S+6}$$

$$Y_{1(S)} = \frac{6/7}{(S+6)^2} \quad ; \quad \text{SEA: } F_{(S)} = \frac{1}{S^2}$$

ENTONCES: $Y_{1(S)} = 6/7 \cdot F_{(S+6)}$

$$y_{1(t)} = \frac{6}{7} e^{-6t} \cdot ramp(t)$$

EN DEFINITIVA:

$$y_{(t)} = 1/4\, e^{-t} \mu(t) + \frac{6}{7} e^{-6t} ramp(t) + \cdots$$
$$+ \cdots \frac{19}{14} e^{-6t} \cdot \mu(t)$$