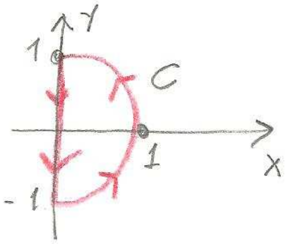
# EVALUACION Nº 3

1) CALCULAR LA INTEGRAL DE LINEA

$$I = \int_C \frac{z}{1+i}\,dz \quad ;$$




Solución:

$$I = \int_{C1} \frac{z}{1+i}\,dz + \int_{C2} \frac{z}{1+i}\,dz = I_1 + I_2$$

$C2 \rightarrow$ LÍNEA RECTA

$$C_1 \Rightarrow \begin{cases} x = \cos\tau \Rightarrow dx = -\operatorname{sen}\tau\,d\tau \\ y = \operatorname{sen}\tau \Rightarrow dy = \cos\tau \,.\, d\tau \end{cases}$$

↳ MEDIA CIRCUNFERENCIA

$$P_0 = \begin{cases} x = 0 \\ y = -1 \end{cases} \Rightarrow \begin{cases} \tau = \cos^{-1}(x) = \pm \pi/2 = -\pi/2 \\ \tau = \operatorname{sen}^{-1}(y) = -\frac{\pi}{2} \end{cases}$$

$$P_f = \begin{cases} x = 0 \Rightarrow \tau = \cos^{-1}(0) = \pm \pi/2 = \pi/2 \\ y = 1 \Rightarrow \tau = \operatorname{sen}^{-1}(1) = \pi/2 \end{cases}$$

SE RESOLVERÁ EN FUNCIÓN DE X

$$I_1 = \int_{C1} \frac{z}{1+i}\,dz = \sqrt{2}\angle{-45^\circ} \int_{C1} (x+iy)\,dz$$

$$I_1 = \overbrace{\sqrt{2}\angle{-45^\circ}}^{K} \left\{ \int_{C1} x\,dx + \hat{i} \int_{C1} y\,dy \right\}$$

$$I_1 = K \left\{ \int_{-\pi/2}^{\pi/2} \cos\tau(-\operatorname{sen}\tau)\,d\tau + \hat{i} \int \operatorname{sen}\tau(\cos\tau)\,d\tau \right.$$

$$I_1 = K \left\{ -\int_{-\pi/2}^{\pi/2} \cos\tau . \operatorname{sen}\tau\,d\tau + \hat{i} \int_{-\pi/2}^{\pi/2} \operatorname{sen}\tau . \cos\tau\,d\tau \right.$$

$u = \operatorname{sen}\tau \Rightarrow du = \cos\tau\,d\tau$

ENTONCES:

$$I = K \left\{ -\int u\,du + \hat{i} \int u \,.\, du = K \left\{ \frac{-u^2}{2} + i\frac{u^2}{2} \right\} \right.$$

ENTONCES:

$$I_1 = K \left\{ -\frac{\operatorname{sen}^2\tau}{2} + i\frac{\operatorname{sen}^2\tau}{2} \right\}_{-\pi/2}^{\pi/2} = K\{0 + i0\}$$

$$I_1 = (\sqrt{2}\angle{-45^\circ})(0) = 0$$

$$*I_2 = K \left\{ \int_{C_2} x\,dx + i \int_{C_2} y\,dy \right\}$$

$C_2 \Rightarrow x = 0 \Rightarrow$ PARAMETRIZANDO $\Rightarrow$

$$\Rightarrow C_2 = \begin{cases} x = 0 \Rightarrow dx = 0 \\ y = \tau \Rightarrow dy = d\tau \end{cases}$$

POR LO TANTO:

$$I_2 = K \int_1^{-1} (0)(0) + i K \int_1^{-1} \tau . d\tau = i K \left[ \frac{\tau^2}{2} \right]_1^{-1}$$

$$I_2 = (\sqrt{2}\angle{-45^\circ})(0) = 0$$

POR LO TANTO:

$$I = 0 + 0 \Rightarrow$$

$$\boxed{I = 0}$$

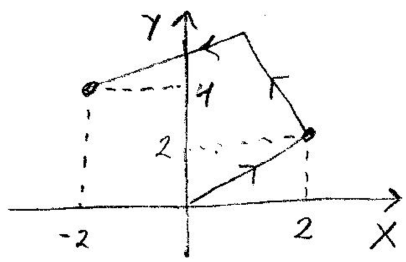
COMO ERA DE ESPERARSE YA QUE $f(z)$ ES CONSERVATIVO Y $C$ ES CERRADA

Prof. Ander Miranda

2) CALCULE EL TRABAJO INDICADO

$$W = \int_C \vec{F} \cdot d\vec{R}$$

DONDE: $\vec{F}(x,y) = \overbrace{(xy^2 + x^2)}^{M}\hat{\imath} + \overbrace{(yx^2 + y^2 + 1)}^{N}\hat{\jmath}$




Solución:

$$M_y = \frac{\partial}{\partial y}(xy^2 + x^2) = 2xy$$

$$N_x = \frac{\partial}{\partial x}(yx^2 + y^2 + 1) = 2xy$$

SON IGUALES CAMPO CONSERVATIVO

ENTONCES:

$$\vec{\nabla}\phi = \underbrace{(xy^2 + x^2)}_{f_x}\hat{\imath} + \underbrace{(yx^2 + y^2 + 1)}_{f_y}\hat{\jmath}$$

$$\int f_x \, dx = \frac{x^2 y^2}{2} + \frac{x^3}{3} + g(y) = f(x,y) \quad \text{I}$$

DERIVANDO CON RESPECTO A Y:

$$f_y = \cancel{x^2 y} + g'(y) = yx^2 + y^2 + 1 \Rightarrow$$

$$g'(y) = y^2 + 1 \Rightarrow g(y) = \frac{y^3}{3} + y \quad \text{II}$$

SUST II EN I

$$f(x,y) = \frac{x^2 y^2}{2} + \frac{x^3}{3} + \frac{y^3}{3} + y$$

$$W = \left. \frac{x^2 y^2}{2} + \frac{x^3}{3} + \frac{y^3}{3} + y \right|_{(0,0)}^{(-2,4)}$$

$$W = 164/3$$

3) CALCULAR $f(x,y)$ SI

$$\vec{\nabla} f(x,y) = (x^3 + 3x)\hat{\imath} + \underbrace{(y^3 + 1)}_{f_y}\hat{\jmath}$$

Solución:

$$f_x = x^3 + 3x \quad \Rightarrow \text{INTEGRANDO}$$

$$\int f_x \, dx = f(x,y) = \frac{x^4}{4} + \frac{3}{2}x^2 + g(y) \quad \text{I}$$

DERIVANDO CON RESPECTO A Y

$$f_y = 0 + g'(y) = y^3 + 1 \Rightarrow$$

$$g'(y) = y^3 + 1 \quad \Rightarrow \text{INTEGRANDO}$$

$$g(y) = \frac{y^4}{4} + y \quad \text{SUST EN I}$$

$$f(x,y) = \frac{x^4}{4} + \frac{3x^2}{2} + \frac{y^4}{4} + y$$